

proached and the apparent effect of pressure in increasing the viscosity increases toward a limiting value. Thus, when at 500° the mean velocity was reduced from 0.5 cm/sec. to 0.1 cm/sec., the apparent pressure coefficient was raised about 25 percent. A further reduction of the velocity to 0.06 cm/sec. gave no perceptible change in the pressure coefficient, within the rather large probable error, amounting to several percent. It seems unlikely that the value of the pressure coefficient for infinitely slow flow will be very much greater than is found with our finest capillary (40 cm long, 0.041 cm diameter). At 360°, the heating effect is certainly negligible.

No corrections have been applied for the compressibility of the glass, which is not known, nor for the elastic stretch of the capillary. These corrections are much smaller than the probable error due to the other causes which have been discussed. The absence of any appreciable creep or permanent stretch of the capillary was verified by taking measurements alternately at high and at low pressures. It is hardly necessary to add that these velocities of flow are far below the Reynolds critical velocity, by a factor of about  $10^9$  at 500°. The shearing stress in the glass, at the wall of the capillary where it is greatest, is about 250 g/cm<sup>2</sup> for the longest capillary, and about 3100 g/cm<sup>2</sup> for the shortest and widest, with an inlet pressure of 1000 kg/cm<sup>2</sup>.

### 3. RESULTS

With this type of viscometer, the pressure to which the glass is subjected decreases from a high value  $P$  at the inner end of the capillary to one atmosphere, which we take as zero, at the outer end. If the viscosity varies with the pressure, and is given by  $\eta(p)$ , the rate of flow  $Q$ , in cm<sup>3</sup>/sec., will be given by the integral

$$Q = C \int_{p=0}^{p=P} dp / \eta(p),$$

where  $C$  is a constant for a given capillary. The viscosity as a function of pressure may be obtained by taking the slope of a plot of  $Q$  as function of the inner pressure  $P$ , as shown by Hersey and Snyder. Another way of treating the results has been adopted here. If the viscosity were independent of the pressure, a plot of the

quantity  $Q/P$  against  $P$  would be a straight horizontal line. On the other hand, if  $\eta$  increases with pressure, the values of  $Q/P$  will decrease as  $P$  increases, and the form of variation of  $\eta$  with the pressure may be derived from such a curve.

In Fig. 2 is shown a representative set of

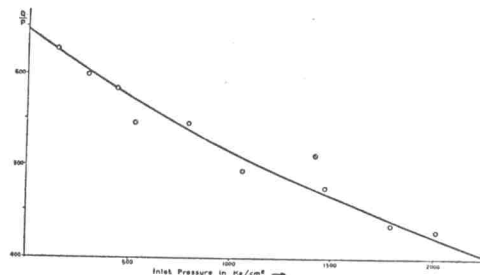


FIG. 2. Experimental results at 516° C.  $Q/P$  in arbitrary units. The smooth curve is calculated from the expression,  $\eta = \eta_0 e^{0.00046p}$ .

points at 516°. This set in which the greatest inlet pressure, about 2000 kg/cm<sup>2</sup>, was attained, is typical of a number of runs at this temperature, with regard to the average slope and to the consistency of the results. The average deviation of the points from any one of several smooth curves is about 2 percent of the ordinate. On the basis of these measurements alone, it is not possible to distinguish among the several simple analytical expressions for the variation of viscosity with pressure which suggest themselves. In the case of the results for 359°, shown in Fig. 3, the measured effect is much greater, the scattering is less, and the choice of expressions is consequently more restricted.

In Table I are given several simple expressions

TABLE I.

| T °C | $\eta/\eta_0$                                                                          | $Q/P$                | VALUE OF COEFFICIENT IN CM <sup>2</sup> /KG | $\eta_{1000}/\eta_0$ |
|------|----------------------------------------------------------------------------------------|----------------------|---------------------------------------------|----------------------|
| 516° | $\left\{ \begin{array}{l} 1/(1-\beta p) \\ (1+ap) \\ e^{\alpha p} \end{array} \right.$ | $A(1-\beta P/2)$     | $\beta = 0.00035$                           | 1.54                 |
|      |                                                                                        | $A \ln(1+ap)/aP$     | $a = 0.00053$                               | 1.53                 |
|      |                                                                                        | $A(1-e^{-\alpha P})$ | $\alpha = 0.00046$                          | 1.58                 |
| 359° | $e^{\alpha p}$                                                                         |                      | $\alpha = 0.0015$                           | 4.48                 |

with numerical values for the coefficients to fit the data for 516°, and one expression for the data at 359°. In addition to the formula for  $\eta(p)$  we give the integrated expression for  $Q/P$ , and the ratio  $\eta_{1000}/\eta_0$  of the viscosity at 1000